1. The functions f and g are defined by

$$f(x) = 7 - 2x^2 \qquad x \in \mathbb{R}$$

$$g(x) = \frac{3x}{5x-1} \qquad x \in \mathbb{R} \qquad x \neq \frac{1}{5}$$

(a) State the range of f

(1)

(b) Find gf(1.8)

(2)

(c) Find $g^{-1}(x)$

(2)

a) $f(x) = 7 - 2x^2$

$f(x) \leq 7$ ①

$-2x^2 \leq 0 \ \forall x$

so max $f(x)$ is when $-2x^2 = 0$

b) $gf(1.8)$

$f(1.8) = 7 - 2(1.8)^2 = 0.52$

$g(0.52) = \dfrac{3(0.52)}{5(0.52)-1}$ ① $= 0.975$ ①

c) $g(x) = \dfrac{3x}{5x-1}$

swap x and y, rearrange for y

$x = \dfrac{3y}{5y-1}$

$x(5y-1) = 3y$

$5xy - x = 3y$

$5xy - 3y = x$

$y(5x-3) = x$ ①

$y = \dfrac{x}{5x-3}$

$g^{-1}(x) = \dfrac{x}{5x-3}$ ①

2. The function f is defined by

$$f(x) = \frac{8x+5}{2x+3} \qquad x > -\frac{3}{2}$$

(a) Find $f^{-1}\left(\frac{3}{2}\right)$

(2)

(b) Show that

$$f(x) = A + \frac{B}{2x+3}$$

where A and B are constants to be found.

(2)

The function g is defined by

$$g(x) = 16 - x^2 \qquad 0 \leqslant x \leqslant 4$$

(c) State the range of g^{-1}

(1)

(d) Find the range of fg^{-1}

(3)

a) $f(x) = \frac{8x+5}{2x+3}$, $x > -\frac{3}{2}$

$y = \frac{8x+5}{2x+3}$

$y(2x+3) = 8x+5$

$2xy + 3y = 8x+5$

$2xy - 8x = 5 - 3y$

$x(2y-8) = 5-3y$

$x = \frac{5-3y}{2y-8}$

Now, swap y with x .

$$y = \frac{5-3x}{2x-8}$$

$$\therefore f^{-1}(x) = \frac{5-3x}{2x-8}$$

$$f^{-1}\left(\tfrac{3}{2}\right) = \frac{5-3\left(\frac{3}{2}\right)}{2\left(\frac{3}{2}\right)-8} \quad (1)$$

$$f^{-1}\left(\tfrac{3}{2}\right) = -\frac{1}{10} \quad (1)$$

b) $$\frac{8x+5}{2x+3} \equiv A + \frac{B}{2x+3}$$

$$8x+5 = A(2x+3) + B \quad (1)$$

$$8x+5 = 2Ax + 3A + B$$

$2Ax = 8x$ $\quad$ $5 = 3A + B$

$2A = 8$ $\quad$ $5 = 3(4) + B$

$A = 4$ $\quad$ $B = -7$

$$\therefore f(x) = 4 - \frac{7}{2x+3} \quad (1)$$

c) $g(x) = 16 - x^2 \quad , \quad 0 \leqslant x \leqslant 4$

range of inverse function is domain of function

$\therefore \quad 0 \leqslant g^{-1}(x) \leqslant 4$ (1)

d) $f(0) = \dfrac{8(0)+5}{2(0)+3} = \dfrac{5}{3}$ (1)

$f(4) = \dfrac{8(4)+5}{2(4)+3} = \dfrac{37}{11}$ (1)

$\therefore \quad \dfrac{5}{3} \leqslant fg^{-1}(x) \leqslant \dfrac{37}{11}$ (1)

3. The function f is defined by

$$f(x) = 3 + \sqrt{x - 2} \qquad x \in \mathbb{R} \quad x > 2$$

(a) State the range of f

(1)

(b) Find f^{-1}

(3)

The function g is defined by

$$g(x) = \frac{15}{x - 3} \qquad x \in \mathbb{R} \quad x \neq 3$$

(c) Find gf(6)

(2)

(d) Find the exact value of the constant a for which

$$f(a^2 + 2) = g(a)$$

(2)

a) $\sqrt{x-2} > 0 \;\; \forall x > 2 \;\; \therefore \; f(x) > 3$ ① $\forall$ means "for all"

b) $f(x) = 3 + \sqrt{x-2}$ swap x and y, rearrange for y

$x = 3 + \sqrt{y-2}$ ①

$x - 3 = \sqrt{y-2}$

$y - 2 = (x-3)^2$

$y = (x-3)^2 + 2$ range of f is the domain of f^{-1}

$f^{-1}(x) = (x-3)^2 + 2$ ①, $x > 3$ ①

c) $f(6) = 3 + \sqrt{6-2} = 3 + 2 = 5$ ①

$g(5) = \frac{15}{5-3} = \frac{15}{2}$ ①

d) $f(a^2+2) = g(a)$

$$3 + \sqrt{a^2+2-2} = \frac{15}{a-3}$$

$$3 + a = \frac{15}{a-3}$$

$$(a+3)(a-3) = 15$$

$$a^2 - 9 = 15 \quad (1)$$

$$a^2 = 24$$

$$a = \pm 2\sqrt{6}$$

$a = -2\sqrt{6}$ doesnt work in original equation

$$a = 2\sqrt{6} \quad (1)$$